



# MULTIVALUED SOLUTIONS OF A NONLINEAR BOUNDARY PROBLEM

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When describing the motion of a particle in a continuously deformed medium (a continuous polaron), one encounters two limit cases where the exact solution is possible. The first case is that of a weak particle-medium interaction which can be considered by the classical theory of perturbations. In the second case of an extremely strong interaction this theory is not applicable. Here the wave function of a particle is described by means of a nonlinear boundary problem. We have considered the following examples:

a) the problem of an electron in a homeopolar crystal (condenson):

$$\Delta y + y^3 - y = 0, \quad y(\infty) = y'(0) = 0$$

b) the problem of an electron in a polar crystal (polaron):

$$\begin{aligned} \Delta y + zy - y &= 0, & \Delta z + y^2 &= 0 \\ y(\infty) = y'(0) &= 0, & z(\infty) = z'(0) &= 0 \end{aligned}$$

c) the problem of an electron in a polar crystal localized in the vicinity of a hydrate-type defect (F-centre):

$$\begin{aligned} \Delta y + zy - Ny/x - y &= 0, & \Delta z + y^2 &= 0 \\ y(\infty) = 2y'(0) + Ny(0) &= 0, & z(\infty) = z'(0) &= 0 \end{aligned}$$

A very important peculiarity of these problems is that their solutions are multivalued. It is shown that for a spheric symmetry the solutions are as follows:  $y_0(x)$  has no zeros (the main state),  $y_1(x)$  has one zero (the first excited state) etc.